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Transient response of multipass plate heat exchangers with axial thermal dispersion in fluid

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Abstract

To predict the transient response of multipass plate heat exchanger, an analysis is presented based on an axial heat dispersion model in the fluid which takes deviation from ideal plug flow into consideration. The method can be utilised to simulate the exit temperatures for both the fluids for any given arbitrary temperature transient at inlets. The methods takes the 'phase-lag' inside the distribution port into consideration and analyse the multipass plate heat exchanger by considering it as an assembly of single pass plate heat exchangers. The solution technique for equal number of passes on both the sides is different from that for unequal number of passes on the two sides. Examples of computation for the response of 1–2 and 2–2 pass configuration have been presented. The 1–2 pass exchanger is solved by analysing successive modules of the heat exchanger. The 2–2 pass heat exchanger is required to be simulated by iterating the responses of two smaller heat exchangers to which it has been divided. The method involves a Laplace transformation with the temporal variable, solution in space domain and numerical inversion of Laplace transform back to the time domain. The results indicate the strong effect of axial dispersion in fluid on the transient response. The effect of number of plates and number of transfer unit has also been studied which brings out a comprehensive picture of responses of a multipass plate heat exchanger in transient regime.

1. Introduction

In last two decades or so, the most important and perceptible change in the heat exchanger industry was the advent of plate and spiral heat exchangers for process, power and heat recovery applications. The plate heat exchangers were originally designed for hygienic applications such as in the dairy and brewing industries to overcome the problem of cleaning and maintenance which is one of the major requirements of such industries. Such exchangers were not used in process and power applications primarily because of the leakage problem in gaskets. Also the gradual deterioration of elastomer gaskets limited them from being used for long performance as in the case of process industries. However, the emergence of new materials and better designs of gaskets have overcome many of these problems and has virtually created a new wave of replacing shell and tube heat exchangers by plate heat exchangers [1]. The factors which attracted this generation of heat exchangers are their compactness, ability to generate higher turbulence at comparatively lower flow rates, smaller hold up volume and hence quicker response to control operations, less amount of flow induced vibrations, much low level of fouling and flexibility in operations.

The literature available for plate heat exchangers

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Nomenclature

A	heat transfer area per effective plate (m^2)
A	coefficient matrix for system of differential equations
A_c	free flow area in channel (m^2)
$A_{k,m}$	the element of kth row and m the column matrix of A
A_w	cross sectional area of plate available for longitudinal conduction (m^2)
B	diagonal matrix, Eq. (28)
C	heat capacity of resident fluid(s) (J K^{-1})
C_p	specific heat of the fluid (J kg^{-1} K^{-1})
C_w	heat capacity of plate material(J K^{-1})
D	axial dispersion coefficient (W m^{-1} K^{-1})
D	matrix resulting from boundary condition
d_j	elements of matrix **D**
$f(Z)$	inlet temperature function
$F(s)$	Laplace transform of $f(Z)$
G	transfer function
h	heat transfer coefficient (W m^{-2} K^{-1})
i	square root of -1
l_i	path traversed by fluid particle before entering ith channel (m)
L	fluid flow length in channels (m)
m_j	mass flow rate of the fluid (kg s^{-1}) $j-2[j/2]$, where j is an integer
m, n	number of passes
n_1	number of odd channel
n_2	number of even channel
N	number of channels
Ntu	number of transfer units $((1/U_1)+(n_1/n_2)(1/R_2U_2))$
Pe	axial dispersive Peclet number, $\dot{w}L/A_cD$
R_2	capacity rate ratio in channels, $\dot{w}_2/\dot{w}_1$
R_{g2}	capacity rate ratio in combined flow, $\dot{w}_{g2}/\dot{w}_{g1}$
$R_{g\tau}$	characteristic time ratio of the combined flow, τ_{rg2}/τ_{rg1}
R_N	ratio U_2/U_1
R_{Pe}	ratio of Peclet numbers in channels, Pe_2/Pe_1
R_w	wall heat capacity ratio, C_w/C_1
R_τ	characteristic time ratio in channels, τ_{r2}/τ_{r1}
s	transformed time variable in Laplace domain
S	matrix with inlet fluid function
t	dimensionless temperature, theta, $\frac{\theta-\theta_{g1,\,in}}{\theta_{g2,\,in}-\theta_{g1,\,in}}$
T	temperature obtained by Laplace transformation of temperature, t, $T=L(t)$
$\boldsymbol{T}$	temperature matrix, in Eq. (28)
u	a unit step function
U	matrix of eigenvectors of the matrix **A**
$u_{i,j}$	elements of **U** matrix
$U_{1(2)}$	$(hA/\dot{w})_{1(2)}$
V_i	the velocity of fluid in gasket port after ith channel (m s^{-1})
V_g	volume flow rate (m^3 s^{-1})
$\dot{w}$	thermal capacity rate of fluid in channels $=\dot{m}C_p$ (W K^{-1})
$\dot{w}_g$	thermal capacity rate of combined fluid $=\sum_{i=1}^{N} w_i$ (W K^{-1})
X	space coordinate (m)
x	dimensionless space coordinate, X/L
Z	dimensionless time, τ/τ_{r1}

Greek symbols

β_j	jth eigenvalue of matrix **A**
γ_w	wall conduction parameter, $A_w\lambda_w/\dot{w}_1L$
θ	temperature (K)
τ	time (s)
Δ_τ	time of travel in port (between channels) (s)
λ_w	thermal conductivity of plate in longitudinal direction (W/m K)
τ_r	resident time $=C/\dot{w}$ (s)
ϕ	dimensionless phase lag (cumulative value)
$\Delta\phi$	dimensionless phase lag (discrete value), $\Delta\tau_i/\Delta\tau_{r1}$

Subscripts

exit	at exit
g	combined flow before splitting on channels or after recombination at exist of ith channel
in	inlet
w	plate
wi	ith plate
0	initial
1	the fluid in odd channels
2	the fluid in even channels

is very vast. The mathematical modelling for plate heat exchangers was presented with the help of numerical scheme by Watson et al. [2]. The subsequent studies on computer simulation of plate heat exchangers were presented by Jackson and Troupe [3] and Marano and Jechura [4]. Simultaneously analytical solutions were presented by Wolf [5], Bounopane et al. [6] and Zeleski [7]. However, all these analyses are of steady state performance which were finally presented in much precise form by Kandlikar and Shah [8]. It is interesting to note that though considerable study has been made about the transient performance of shell and tube heat exchangers, similar analyses for plate heat exchangers are extremely scarce. McKnight and Worley [9] were the first to bring out study in this area. They used the feed back control of high velocity flow for this performance analysis. Zeleski and Tajszerki [10] presented the simulation of dynamic performance for co-current plate heat exchangers. Lakshmanan and Potters [11] used their 'cinematic model' to predict the dynamic behaviour of single pass plate heat exchangers numerically. Khan et al. [12] presented the experimental and analytical studies of counter current plate heat exchangers using sinusoidal and pulse inputs. These few transient studies available in literature depict only the single pass plate heat exchangers. The only study into the dynamics of multipass plate heat exchangers is from Masubuchi and Ito [13]. This study again suffers from the simplified assumption of plug flow in fluid and absence of phase lag between the successive channels.

The present analysis aims to develop a transient simulation of multipass plate heat exchangers with axial dispersion of heat in fluid and phase lag effect. Analysis of single pass plate heat exchangers with similar axial dispersion in fluid which takes care of deviation from plug flow (flow maldistribution and back mixing) and phase lag effect to account for delay at the distribution port was first studied by Das and Roetzel [14]. The scope of the present study has been limited to the two most popular and simple multipass configurations. However, the method presented here can be extended to any pass arrangement. In this paper the governing differential equation has been derived for parallel and counter flow plate heat exchangers separately. The actual multipass heat exchangers have been divided into single pass modules, the solution for which has been obtained using the method of Laplace transform. The transformed equation has been solved using an eigen system analysis, taking Danckwert [15] boundary conditions into consideration. For 1–*n* type pass arrangement the solution can be obtained by obtaining response from successive heat exchanger modules (parallel or counter flow) independently. However, for multiple number of passes on both the sides this approach fails due to conjugate nature of the problem and hence, it has to be solved by an iterative method between the successive modules. The results are presented to depict the effect of axial dispersion. It is important to mention that multipass plate heat exchangers which are widely used for industrial applications either to encounter unequal capacity rates on both sides or to utilise the allowable pressure drop of each fluid has hardly been studied for transient regime which is critical for control strategies. The method presented here can be developed as a general method for transient analysis of all types of multipass heat exchangers.

2. Mathematical formulation

The multipass plate heat exchangers are modelled mathematically by resorting to the following assumptions which are reasonable for normal range of operation with temperature transients. The axial heat dispersion in fluid which accounts for deviation of fluid flow from the plug flow model has been one important assumption which has been confirmed by previous study [13].

Assumptions

1. All flow and thermal properties are independent of temperature.
2. The flow velocity and the mean heat transfer coefficient are uniform over channel length and are identical for the channels carrying similar fluids but different for dissimilar fluids.
3. Thermal conductance is infinite across the width and thickness of plate but negligible along plate length.
4. Heat transfer takes place only across the plates and not through ports, sealing edges and gaskets.
5. The heat exchanger is insulated from heat leak to the atmosphere.
6. The fluid is mixed in the port between the successive passes.
7. The flow maldistribution in the flow passages and the temperature distribution across (perpendicular flow direction) the channel can be described by introducing an axial dispersion term into the energy equation. The dispersion coefficient being a flow property.
8. The exchanger is started from cold (atmospheric) condition.

It must be mentioned here that the consequence of assumption 7 is that — we can now treat the flow within the channel as a plug flow with an extra dispersion term which takes care of any deviation from plug flow such as flow maldistribution or temperature distribution across the channel. Thus, we can say

$$\begin{aligned}&\text{Actual flow and temperature distribution}\\&\quad=\text{Plug flow with uniform temperature across the channel}\\&\quad+\text{A dispersion term to account for the deviation from}\\&\quad\text{the plug flow}\end{aligned}$$

It may be mentioned here that treatment of the problem with a plug flow does not introduce any error since the deviations from the plug flow are taken care by the dispersion coefficient as has been shown in the previous analyses [14,16].

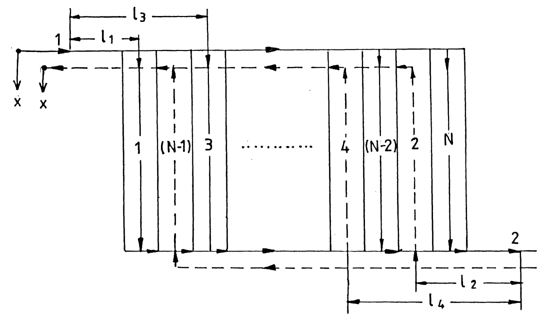


Fig. 1. Schematic diagram for Z-type counter flow plate heat exchangers.

3. Single pass plate heat exchangers

The counter flow single pass plate heat exchangers can be shown as in Fig. 1 and the parallel flow as in Fig. 2. For both of these heat exchangers, the coordinate systems are chosen from top to bottom direction even though this choice is quite arbitrary. For both the configurations, the channels are named from 1 to N and the plates from 1 to $N+1$. The direction of fluid flow at the Nth channel will obviously depend on total number of channels (which is chosen to be odd). Furthermore, cold fluid is assumed to flow to the two extreme channels as this is a common practice to minimise the heat loss to the surroundings. However, it should be mentioned here that these considerations are not essential for the simulation and it will be seen later that to analyse multipass plate heat exchangers we may have to analyse single pass plate heat exchangers with even number of channels.

Before deriving the governing equations, it is to be noted that all the plates are wetted by two different fluids on two sides excepting the two end plates which are in contact with only one fluid. The sides of these two plate which are open to environment are assumed to be insulated. With axial heat dispersion in both the fluid an energy balance over differential element in

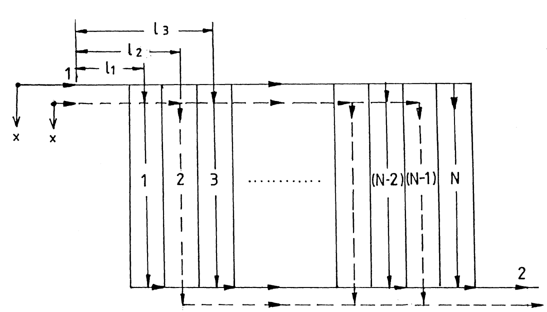


Fig. 2. Schematic diagram for Z-type parallel flow plate heat exchangers.

channel and plate yields the following governing differential equations.

Equation for plates

$$\frac{C_{\mathrm{w}}}{L}\frac{\partial\theta_{\mathrm{w}i}}{\partial\tau}=\lambda_{\mathrm{w}}A_{\mathrm{w}}\frac{\partial^{2}\theta_{\mathrm{w}i}}{\partial X^{2}}+\frac{(hA)_{1}}{2L}(\theta_{i-1}-\theta_{\mathrm{w}i})+\frac{(hA)_{2}}{2L}(\theta_{i}-\theta_{\mathrm{w}i}) \tag{1}$$

$$i=2,4,6,\ldots,2\left[\frac{N+1}{2}\right]$$

$$\frac{C_{\mathrm{w}}}{L}\frac{\partial\theta_{\mathrm{w}i}}{\partial\tau}=\lambda_{\mathrm{w}}A_{\mathrm{w}}\frac{\partial^{2}\theta_{\mathrm{w}i}}{\partial X^{2}}+\frac{(hA)_{2}}{2L}(\theta_{i-1}-\theta_{\mathrm{w}i})+\frac{(hA)_{1}}{2L}(\theta_{i}-\theta_{\mathrm{w}i}) \tag{2}$$

$$i=3,5,7,\ldots,2\left[\frac{N}{2}\right]-1$$

$$\frac{C_{\mathrm{w}}}{L}\frac{\partial\theta_{\mathrm{w}1}}{\partial\tau}=\lambda_{\mathrm{w}}A_{\mathrm{w}}\frac{\partial^{2}\theta_{\mathrm{w}i}}{\partial X^{2}}+\frac{(hA)_{1}}{2L}(\theta_{1}-\theta_{\mathrm{w}1}) \tag{3}$$

$$\frac{C_{\mathrm{w}}}{L}\frac{\partial\theta_{\mathrm{w}n-1}}{\partial\tau}=\lambda_{\mathrm{w}}A_{\mathrm{w}}\frac{\partial^{2}\theta_{\mathrm{w}n-1}}{\partial X^{2}}+\frac{(hA)_{*}}{2L}(\theta_{n}-\theta_{\mathrm{w}n-1}) \tag{4}$$

Equation for fluids

(a) For counter flow

$$\frac{C_{1}}{L}\frac{\partial\theta_{i}}{\partial\tau}=A_{\mathrm{c}}D_{1}\frac{\partial^{2}\theta_{i}}{\partial X^{2}}-(-1)^{i-1}\dot{w}_{1}\frac{\partial\theta_{i}}{\partial X}+\frac{(hA)_{1}}{2L}\left(\theta_{\mathrm{w}i}+\theta_{\mathrm{w}_{i-1}}-2\theta_{i}\right) \tag{5}$$

$$i=1,3,\ldots,2\left[\frac{N+1}{2}\right]-1$$

$$\frac{C_{2}}{L}\frac{\partial\theta_{i}}{\partial\tau}=A_{\mathrm{c}}D_{2}\frac{\partial^{2}\theta_{i}}{\partial X^{2}}-(-1)^{i-1}\dot{w}_{2}\frac{\partial\theta_{i}}{\partial X}+\frac{(hA)_{2}}{2L}\left(\theta_{\mathrm{w}i}+\theta_{\mathrm{w}_{i-1}}-2\theta_{i}\right) \tag{6}$$

$$i=2,4,\ldots,2\left[\frac{N}{2}\right]$$

$$\frac{C_1}{L}\frac{\partial\theta_i}{\partial\tau}=A_cD_1\frac{\partial^2\theta_i}{\partial X^2}-\dot{w}_1\frac{\partial\theta_i}{\partial X}+\frac{(hA)_1}{2L}\left(\theta_{wi}+\theta_{w_{i-1}}-2\theta_i\right)\tag{7}$$

$$i=1,3,\ldots,2\left[\frac{N+1}{2}\right]-1$$

(b) For parallel flow

$$\frac{C_2}{L}\frac{\partial\theta_i}{\partial\tau}=A_cD_2\frac{\partial^2\theta_i}{\partial X^2}-\dot{w}_2\frac{\partial\theta_i}{\partial X}+\frac{(hA)_2}{2L}\left(\theta_{wi}+\theta_{w_{i-1}}-2\theta_i\right)\tag{8}$$

$$i=2,4,\ldots,2\left[\frac{N}{2}\right]$$

here $(hA)* = (hA)_1$ for odd N; $(hA)* = (hA)_2$ for even N

Here heat conduction along the plate length is found to be of much lower order ($\gamma_w \sim 10^{-6}$) compared to the convective heat transfer because the plate cross section A_w is very small. So the axial heat conduction in plates can be neglected [16].

The large number of variables involved in these equations can be reduced to a very few dimensionless groupings like the number of transfer units Ntu, heat capacity ratio R_2, and Peclet number Pe which are accepted parameters used to characterize heat exchangers. The temperature and spatial co-ordinates can also be reduced to dimensionless form.

Consequently, the governing equations can be reduced to the following non-dimensional equations.

Equations for fluids
(a) For counter flow

$$R_\tau^{m_{i-1}}\frac{\partial t_i}{\partial Z}=\frac{1}{Pe_1(R_{Pe})^{mi-1}}\frac{\partial^2 t_i}{\partial x^2}-(-1)^{i-1}\frac{\partial t_i}{\partial x}+\left(\frac{U_1}{2}R_N^{m_{i-1}}\right)(t_{wi}+t_{wi}-2t_i)\tag{9}$$

$$i=1,2,3,\ldots,N$$

(b) for parallel flow

$$R_\tau^{m_{i-1}}\frac{\partial t_i}{\partial Z}=\frac{1}{Pe_1(R_{Pe})^{m_{i-1}}}\frac{\partial^2 t_i}{\partial x^2}-\frac{\partial t_i}{\partial x}+\left(\frac{U_1}{2}R_N^{m_{i-1}}\right)(t_{wi-1}+t_{wi}-2t_i)\tag{10}$$

$$i=1,2,3,\ldots,N$$

Wall equations (for both parallel and counter flow)

$$R_w\frac{\partial t_{wi}}{\partial Z}=\frac{U_1}{2}(R_N\cdot R_2)^{m_i}(t_{i-1}-t_{wi})+\frac{U_1}{2}(R_N\cdot R_2)^{m_{i-1}}(t_i-t_{wi})\tag{11}$$

$$i=2,3,\ldots,N$$

$$R_w\frac{\partial t_{wi}}{\partial Z}=\frac{U_1}{2}(t_1-t_{w1})\tag{12}$$

$$R_w\frac{\partial t_{wN-1}}{\partial Z}=\frac{U_1}{2}(R_N\cdot R_2)^{m_{N-1}}(t_w-t_{wN-1})\tag{13}$$

3.1. The boundary conditions

The boundary conditions associated with plate heat exchangers are rather complicated due to the existence of 'phase-lag effect' as discussed by Das and Roetzel [14]. During transient operation the temperature change do not enter all the channels at the same time. The fluid enters channel 1, 2, 3,... with an increasing phase lag from the time at which the combined flow enters the heat exchanger at point 1 (Figs. 1 and 2) In a U-type plate heat exchanger this effect gets even more increased, because the fluids from channels 1, 2, 3,... also encounter increasing amount of time delay before they mix up and the combined stream reaches the exit point 2. In a Z-type plate heat exchanger the situation is different. Here, unlike a U-type exchanger,

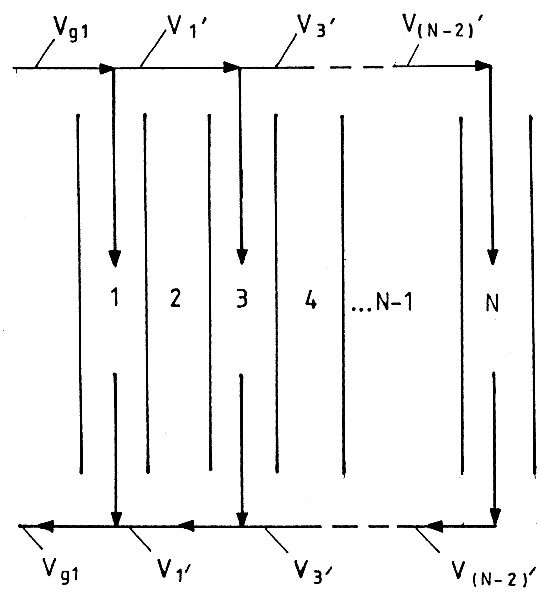


Fig. 3. Velocity change in port during flow distribution.

each fluid stream travels an equal length of path within the heat exchanger. This means that the effect of phase lag at the entry of the channel is reduced by the decreasing phase lag of the steams from channels 1, 2, 3,.... With the assumption of equal flow rate in each channel for a given fluid and uniform inlet and outlet port area of the heat exchanger the phase lag is not only due to plate spacing, but also due to the fact that the fluid velocity in the inlet port decreases as the successive streams leaves the port. This is shown in Fig. 3, where the velocities $V_1, V_2, V_3, \ldots$ are the velocities in the conduit after channels 1, 2, 3,..., respectively. From the continuity condition the ratios of these velocities with the entrance velocity may be derived as

$$\frac{V_{(2i-1)}}{V_{g1}} = 1 - i\frac{\dot{V}_1}{\dot{V}_{g1}} = 1 - \frac{i}{n_1} \quad (\text{for } i = 1, 2, 3, \ldots, n_1) \tag{14}$$

$$\frac{V_{2i}}{V_{g2}} = 1 - i\frac{\dot{V}_2}{\dot{V}_{g2}} = 1 - \frac{1}{n_2} \quad (\text{for } i = 1, 2, 3, \ldots, n_2) \tag{15}$$

The time required for the fluid to travel the distance between the channels can be calculated as (distances shown in Figs. 1 and 2)

$$\Delta\tau_1 = \frac{l_1}{V_{g1}} \tag{16}$$

$$\Delta\tau_2 = \frac{l_2}{V_{g2}} \tag{17}$$

$$\Delta\tau_{2i+1} = \frac{l_{2i+1} - l_{2i-1}}{V_{(2i-1)}} \quad (i = 1, 2, 3, \ldots, (n_1 - 1)) \tag{18}$$

$$\Delta\tau_{2i+2} = \frac{l_{2i+2} - l_{2i}}{V_{2i}} \quad (i = 1, 2, 3, \ldots, (n_2 - 1)) \tag{19}$$

Hence the dimensionless phase lag between the entrance of consecutive channels may be expressed as

$$\Delta\phi_i = \frac{\Delta\tau_i}{\tau_{r1}} \tag{20}$$

The total phase lag at the entry of each channel is the cumulated sum of the phase lags given by

$$\phi_{2i-1} = \sum_{j=1}^{2i-1} \Delta\phi_{2j-1} \quad (i = 1, 2, \ldots, n_1) \tag{21}$$

$$\phi_{2i} = \sum_{j=1}^{2i} \Delta\phi_{2j} \quad (i = 1, 2, \ldots, n_2) \tag{22}$$

The phase lag encountered at the exit of the channels to arrive at the exit point can be computed in a similar way. Under the condition of dimensional symmetry in construction, as shown in Figs. 1 and 2, the relationship for this phase lag at exit reduces to

$$\phi_{i,\,\text{exit}} = \phi_i \tag{23}$$

for a U-type plate exchanger, and

$$\phi_{i,\,\text{exit}} = \phi_{n-i-1} \tag{24}$$

for a Z-type plate exchanger.

The fluid boundary condition at the entry of the channel is of particular interest and attention is to be focussed on it. According to Danckwert [15], a temperature drop should be experienced at the entry section where the dispersion of fluid begins. If the flow in the conduit leading to the heat exchanger channels is plug flow (or flow with very weak dispersion) a sudden drop in the fluid temperature at the entrance of channels, as shown in Fig. 4 will be observed. This gives the following set of boundary conditions in non-dimensional form.

$$t_i - \frac{1}{Pe_1}\frac{\partial t_i}{\partial x} = f_1(Z - \Phi_i)u(Z - \Phi_i) \quad i = 1, 3, 5, \ldots, 2\left[\frac{N+1}{2}\right] - 1 \tag{25}$$

$$\frac{\partial t_i}{\partial x} = 0 \quad i = 2, 4, 6, \ldots, 2\left[\frac{N}{2}\right] \tag{26}$$

(a) For counter flow, at $x = 0$

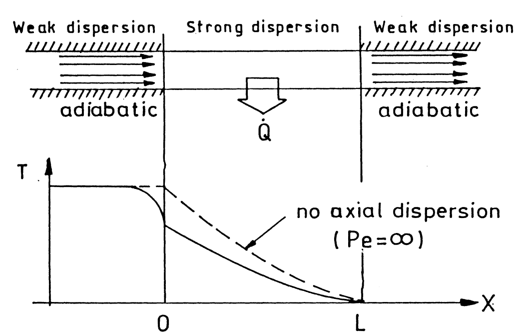


Fig. 4. Effect of axial heat dispersion on the temperature profile at the entrance of a heat transfer apparatus.

$$t_i + \frac{1}{Pe_1} R_{Pe} \frac{\partial t_i}{\partial x} = f_2(Z - \Phi_i)u(Z - \Phi_i)$$
$$i = 2, 4, \ldots, 2\left[\frac{N}{2}\right] \tag{27}$$

at $x = 1.0$

$$\frac{\partial t_i}{\partial x} = 0 \quad i = 1, 3, 5, \ldots, 2\left[\frac{N+1}{2}\right] - 1 \tag{28}$$

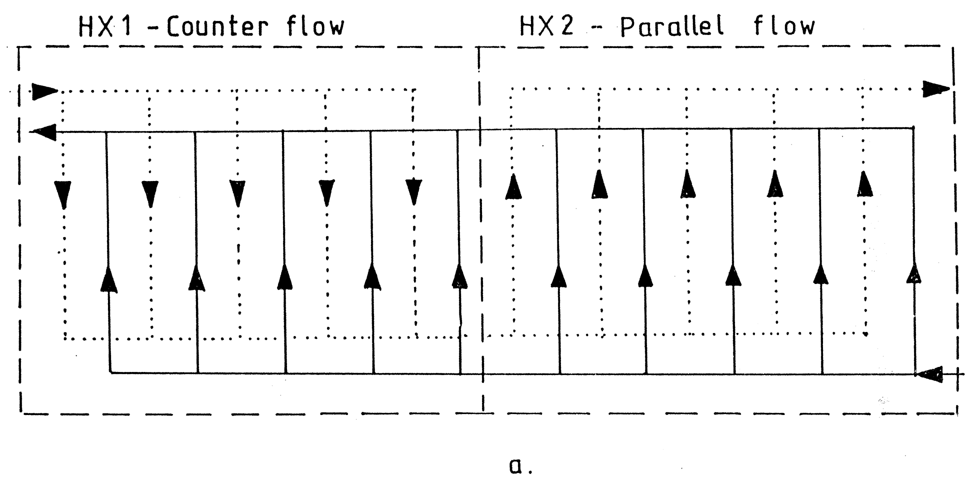
(b) for parallel flow, at $x = 0$

$$t_i - \frac{1}{Pe_1} \frac{\partial t_i}{\partial x} = f_1(Z - \Phi_i)u(Z - \Phi_i)$$
$$i = 1, 3, 5, \ldots, 2\left[\frac{N+1}{2}\right] - 1 \tag{29}$$

$$t_i - \frac{1}{Pe_1 R_{Pe}} \frac{\partial t_i}{\partial x} = f_2(Z - \Phi_i)u(Z - \Phi_i)$$
$$i = 2, 4, 6, \ldots, 2\left[\frac{N}{2}\right] \tag{30}$$

at $x = 1.0$

$$\frac{\partial t_i}{\partial x} = 0 \quad i = 1, 2, 3, \ldots, N \tag{31}$$



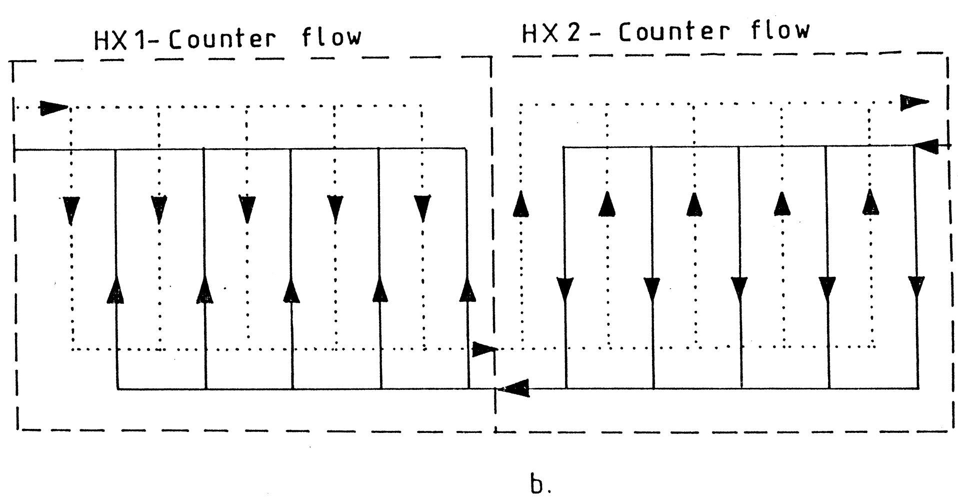


Fig. 5. (a) Schematic diagram of 1–2 pass arrangement. (b) Schemetic diagram of 2–2 pass arrangement.

It is to be noted in the subsequent section, that the wall temperature is eliminated in the coupled equation when plate conduction is negligible. Hence boundary conditions for plates are not required.

4. Multi pass plate heat exchangers

The mathematical formulation presented in the preceding section can be used as a subsystem for the analysis of multipass plate heat exchangers. Broadly two divisions can be made among the multipass plate heat exchangers for the convenience of analysis.

1. 1–*n* type plate heat exchangers
2. *m*–*n* or *n*–*n* type plate heat exchangers

The simplest examples of these two families of plate heat exchangers are 1–2 and 1–2 plate heat exchangers, respectively which are shown in Figs. 5 and 6. These two heat exchangers can be analysed conveniently by splitting them into two different combinations of parallel and counter flow heat exchangers. The divisions are also shown in Figs. 5(a) and 6(b) by dotted lines. It should be noted here this artificial division brings about an element of error at the interface of two heat exchangers by not considering heat interaction there. However, proper correction can be made in this regard by adding a fictitious channel to each heat exchanger module. The mutual interaction of the two smaller heat exchangers (HX1 and HX2) to which the entire heat exchangers has been divided, are shown in Fig. 6(a) and (b).

4.1. Analysis of 1–n type multipass heat exchangers

It is interesting to note from Fig. 6(a) that for the 1–2 heat exchanger all the inputs for HX1 are known and hence its response can be obtained independent of HX2. Consequently, a successive marching type solution can be obtained in which the outlet of cold side temperature of HX1 will act as input to the HX2. This approach can be extended subsequently to 1–*n* type plate heat exchangers. The boundary conditions encountered in such case has to consider that the cold fluid from all the channels get mixed in the port at the outlet of HX1 and gets redistributed in HX2. As far as hot fluid is concerned, it is assumed to have only one pass and it is distributed between channels of HX1 and HX2.

4.2. n–n/m–n type multipass plate heat exchangers

The situation in 2–2 pass plate heat exchangers is shown in Figs. 5(a) and 6(b). The performance of HX2 is influenced by HX1 and vice versa. This transforms it into a conjugate problem between HX1 and HX2 which can be solved only by iterative procedure. The procedure can be conveniently extended to any number of passes for both sides eventhough the computation becomes complex with increasing number of passes.

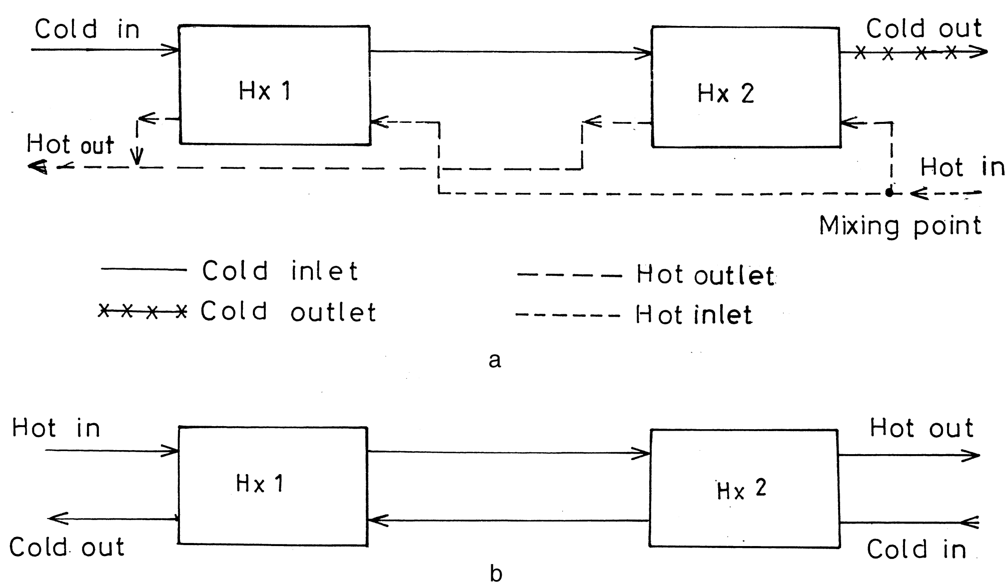


Fig. 6. (a) Block diagram of 1–2 pass arrangement. (b) Block diagram of 2–2 pass arrangement.

5. Solution procedure

5.1. Single pass

The set of partial differential equations for wall expressed by Eqs. (11)–(13) along with the appropriate fluid Eq. (9) or Eq. (10) i.e. Eq. (9) for counter flow and Eq. (10) for parallel flow, constitute the mathematical model for single pass plate heat exchangers subject to the boundary conditions expressed by Eqs. (25)–(28) for counter flow and Eqs. (29)–(31) for parallel flow. The initial condition for both fluid and wall are taken to be uniform. In non-dimensionalising, this initial temperature is set to zero level so as to get

$$t_i(x, Z=0) = t_{\mathrm{w}}(x, Z=0) = 0 \tag{32}$$

The set of governing partial differential equations can be transformed to the following ordinary differential equations by taking Laplace transform with respect to the non-dimensional time variable, Z.

$$T_{\mathrm{w}i} = \frac{\frac{U_1}{2}(R_N R_2)^{m_i} T_{i-1} + \frac{U_1}{2}(R_N R_2)^{m_{i-1}} T_i}{sR_{\mathrm{w}} + \frac{U_1}{2}(R_N R_2)^{m_i} + \frac{U_1}{2}(R_N R_2)^{m_{i-1}}} \tag{33}$$

$$T_{\mathrm{w}-1} = \frac{\frac{U_1}{2}}{R_{\mathrm{w}} s + \frac{U_1}{2}} T_1 \tag{34}$$

$$T_{\mathrm{w}N-1} = \frac{U_1 (R_N R_2)^{m_{N-1}}}{R_{\mathrm{w}} s + \frac{U_1}{2}(R_N R_2)^{m_{N-1}}} \tag{35}$$

It is interesting to note that the absence of longitudinal conductivity term the plate equations reduce to simple algebraic equations.

Substituting these explicit expressions for wall temperatures in the transformed governing equations for fluid.

(a) For counter flow

$$\begin{aligned}\frac{\mathrm{d}^2 T_i}{\mathrm{d}x^2} = Pe R_{Pe}^{m_{i-1}} \Bigg[& R_\tau^{m_{i-1}} \\ & - U_1 R_N^{m_{i-1}} \frac{R_{i-1}}{sR_{\mathrm{w}} + R_i + R_{i-1}} \\ & - \frac{U_1}{2} R_N^{m_{i-1}} \frac{R_{i-1}}{sR_{\mathrm{w}} + R_{i-1} + R_{i-2}} \Bigg] T_i \\ & - \frac{U_1}{2} Pe_1 (R_{Pe} R_N)^{m_{i-1}} \frac{R_i}{sR_{\mathrm{w}} + R_i + R_{i-1}} T_{i-1} \\ & - \frac{U_1}{2} (Pe_1 R_N)^{m_{i-1}} \frac{R_{i-2}}{sR_{\mathrm{w}} + R_{i-1} + R_{i-2}} T_{i-1} \\ & + (-1)^{i-1} Pe_1 R_{Pe}^{m_{i-1}} \frac{\mathrm{d}T_i}{\mathrm{d}x}\end{aligned} \tag{36}$$

(b) For parallel flow

$$\begin{aligned}\frac{\mathrm{d}^2 T_i}{\mathrm{d}x^2} = Pe R_{Pe}^{m_{i-1}} \Bigg[& R_\tau^{m_{i-1}} \\ & - U_1 R_N^{m_{i-1}} \frac{R_{i-1}}{sR_{\mathrm{w}} + R_i + R_{i-1}} \\ & - \frac{U_1}{2} R_n^{m_{i-1}} \frac{R_{i-1}}{sR_{\mathrm{w}} + R_{i-1} + R_{i-2}} \Bigg] T_i \\ & - \frac{U_1}{2} Pe_1 (R_{Pe} R_N)^{m_{i-1}} \frac{R_i}{sR_{\mathrm{w}} + R_i + R_{i-1}} T_{i-1} \\ & - \frac{U_1}{2} (Pe_1 R_N)^{m_{i-1}} \frac{R_{i-2}}{sR_{\mathrm{w}} + R_{i-1} + R_{i-2}} T_{i-1} \\ & - Pe_1 R_{Pe}^{m_{i-1}} \frac{\mathrm{d}T_i}{\mathrm{d}x}\end{aligned} \tag{37}$$

where $m_j = j - 2[j/2]$.

This system of equations can be recast in the matrix form

$$\frac{\mathrm{d}\bar{T}}{\mathrm{d}x} = \bar{A}\bar{T} \tag{38}$$

$$\bar{T} = \left(T_1, T_2, \ldots, T_N, \frac{\mathrm{d}T_1}{\mathrm{d}x}, \frac{\mathrm{d}T_2}{\mathrm{d}x}, \ldots, \frac{\mathrm{d}T_N}{\mathrm{d}x} \right)^{\mathrm{T}}$$

the coefficient $\mathbf{A}$ matrix can be given by

$$A_{n-i,\,i} = Pe_1 R_{Pe}^{m_{i-1}}\left[R_\tau^{m_{i-1}} s + U_1 R_N^{m_{i-1}} - U_1 R_N^{m_{i-1}} \frac{R_{i-1}}{sR_w + R_i + R_{i-1}} - \frac{U_1}{2} R_n^{m_{i-1}} \frac{R_{i-1}}{sR_w + R_{i-1} + R_{i-2}}\right]$$

$i = 1, 2, \ldots, N$

$$A_{n-i,\,i-1} = \frac{U_1}{2} Pe_1 (R_{Pe} R_N)^{m_{i-1}} \frac{R_{i-2}}{sR_w + R_{i-1} + R_{i-2}}$$

$i = 1, 2, \ldots, N-1$

$$A_{N-i,\,N-i} = (-1)^k \cdot Pe \cdot R_{Re}^{m_{i-1}} A_{i,\,N-1} = 1.0$$

$i = 1, 2, \ldots, N$

$$A_{N-i,\,i-1} = \frac{U_1}{2} Pe_1 (R_{Pe} R_N)^{m_{i-1}} \frac{R_i}{sR_w + R_i + R_{i-1}}$$

$i = 2, 3, \ldots, N$

Where $k = i - 1$ for counter flow and 1 for parallel flow, rest of the elements being zero. The solution to Eq. (38) can be written in the form

$$\bar{T} = \bar{U}\bar{B}(x)\bar{D} \tag{39}$$

$\bar{U} = [u_{i,\,j}]$, the matrix of eigenvectors of the coefficient matrix $\bar{A}$

$$\bar{B}(x) = \operatorname{diag}\left(e^{\beta_1,\,x}, e^{\beta_2,\,x}, \ldots,\ e^{\beta_n,\,x}\right)$$

$$\bar{D} = [d_1, d_2, \ldots,\ d_n]^{\mathrm{T}}$$

The individual temperatures and temperature gradients can be written as

$$T_i = \sum_{j=1}^{2N} d_j u_{ij} e^{\beta_j x} \tag{40}$$

$$\frac{\mathrm{d}T_i}{\mathrm{d}x} = \sum_{j=1}^{2N} d_j u_{N+i,\,j} e^{\beta_j x} \tag{41}$$

The coefficient vector can be determined by applying Eqs. (40) and (41) to the Laplace transformed boundary conditions of Eqs. (25)–(28) for counter and Eqs. (29)–(31) for parallel flow to obtain the matrix equation

$$\bar{W}\bar{D} = \bar{S} \tag{42}$$

For example, if we use boundary condition (25) for the 1st channel, with the help of Eqs. (40) and (41) we get the first row of matrix $\bar{W}$ can be written as

$$w_{11} = u_{11} + \frac{u_{N+1}}{Pe_1} \text{ and } w_{12} = u_{12} + \frac{u_{N+2}}{Pe_1} \text{ and so on}$$

where N is the total number of channels.

Obviously **S** the right hand vector contains the inlet temperature functions along with its phase lag, and

$$\bar{S} = \left[F_1(s)e^{-\Phi_1 s}, F_2(s)e^{-\Phi_2 s}, F_1(s)e^{-\Phi_3 s}, F_2(s)e^{-\Phi_4 s}, \ldots, F_k(s)e^{-\Phi_N s}, 0{,}0, \ldots, 0\right]^{\mathrm{T}}$$

where $k = 1$ for odd N; $k = 2$ for even N. Hence D can be derived from

$$\bar{D} = \bar{W}^{-1}\bar{S}$$

6. Response in the time domain

The temperature given by Eq. (29) as calculated by the procedure described in the previous section lies in the frequency domain which has to be mapped to the time domain by Laplace inversion. It is obvious that expressions are too complex to carry out the inversion analytically. Hence Laplace inversion with Crump's [17] numerical algorithm using Fourier series approximation has been used in the present analysis. According to this algorithm, for any function $g(Z)$ with a Laplace transform $G(s)$ it can be expressed as

$$g(Z) = \frac{\exp(aZ)}{Z}\left[\frac{1}{2}G(a) + Re\sum_{k=i}^{\infty} G\left(a + \frac{ik\pi}{Z}\right)(-1)^k\right] \tag{43}$$

The constant a is chosen in the domain $4 < aZ < 5$ to minimize the truncation error. It can be further simplified by using fast Fourier transform. Substituting $Z = 2nZ/M$, the above equation results

$$g(Z_n) = \frac{\exp(aZ_n)}{Z}\left[Re\sum_{k=0}^{M-1} G\left(a + \frac{ik\pi}{Z}\right)x \exp\left(i\frac{2\pi nk}{M}\right) - \frac{1}{2}g(a)\right] \tag{44}$$

The term indicated by summation is obtained by fast Fourier transform at every point Z_n in that domain.

The whole solution process can be summarised as

(i) Transformation of the dimensionless governing

Eqs. (9)–(13) into Laplace plane.
(ii) Framing of the matrix differential Eq. (38) with coefficient matrix $\bar{A}$, from transformed differential equations.
(iii) Obtaining the eigenvalues and eigenvectors of matrix $\bar{A}$.
(iv) Framing of the solutions in the form of Eqs. (40) and (41).
(v) To evaluate the coefficients d_j, using the boundary conditions, the system of linear equations given by Eq. (42) are solved.
(vi) Finally the obtained solutions given by Eqs. (40) and (41) are inverted back to time domain using the algorithm given by Eq. (44).

7. Multipass plate heat exchangers

7.1. 1–2 pass type

The solutions obtained for parallel and counter flow plate heat exchangers following the procedure outlined in the preceding section is used to compute the response of the HX1 of Fig. 5. The resulting cold side outlet temperature is approximated as a first order response [16] characterised by the transfer function.

$$G = K\frac{e^{-\tau_d s}}{1+\tau_c s} \tag{45}$$

where τ_c is time constant. τ_d is delay time, and K is gain.

The values of K, τ_c, τ_d, have been determined by a least square fit to the computed data for temperature response. Subsequently cold side inlet function of heat exchanger HX2 is taken as

$$F_2(s) = \frac{1}{s}xG$$

Hence, the response for HX2 can be calculated for both hot and cold fluids.

The final response of total 1–2 type multipass plate heat exchangers can be obtained by assembling the responses of HX1 and HX2 with proper phase lag effect at the exits of heat exchangers. This implies that the response of the hot side is to be calculated from the mixing of hot side responses of HX1 and HX2 before exit point. As far as cold side response is concerned, it is equal to the cold side response HX2 only, since the cold response of HX1 is only acting as coldside input to the HX2. Thus, the solution procedure reduces to the successive solution of HX modules explicitly and approximating the intermediate responses by proper transfer function. The method can be extended to any 1–n pass plate heat exchanger.

The inaccuracy introduced in the solution by the interface adiabatic condition, resulting at the partition boundary of HX1 and HX2 can be taken care of by adding one additional fictitious channel to each of HX1 and HX2 which compensates for the heat transfer there. However, the effect becomes less significant for increasing number of channels.

7.2. 2–2 pass type

In this type of pass arrangement it is impossible to obtain the responses of either HX1 or HX2 independent of other one and this leads to a conjugate problem. Since, the heat exchangers start from the cold state, it can be observed that the inlet temperatures for cold side is initially zero. Hence, the response for both fluids of the HX1 can be obtained by applying step input on hotside. However, the scenario changes once the HX2 starts responding. Subsequently, the cold side inlet temperature of HX1 changes and hence hot response of the HX1 changes accordingly. The initial hot response of HX1 can be approximated by a first order function used as an input to HX2 hotside and the cold response of HX2 thus calculated is again approximated by first order function and used as cold input to HX1. This procedure of successive iteration is continued until the values of K, τ_c, and τ_d of both hot response of HX1 and cold response of HX2 converge. The final hot and cold responses are given by

$$(T_{\text{out, hot}})_{2\text{–}2\ \text{HX}} = (T_{\text{out, hot}})_{\text{HX2}}$$

$$(T_{\text{out, cold}})_{2\text{–}2\ \text{HX}} = (T_{\text{out, cold}})_{\text{HX1}} \tag{46}$$

The method can be extended to any n–n or n-m pass plate heat exchanger. Here the heat exchanger is divided into few modules of single/multipass heat exchanger and computation can be started with a zero cold inlet temperature at the heat exchanger module to the hot end. An 'iteration sweep' is made upto the cold end to obtain estimates of the intermediate temperatures. The successive sweeps of iterations are carried out till the parameters (K, τ_c, and τ_d) converge for all the intermediate temperature responses.

8. Results and discussions

By applying the procedure mentioned above, the response for all the types of temperature transients can be calculated for change in either of the fluid inlet temperature (or both). As examples some of the results have been computed and presented here. The examples are chosen to bring out the effects of dispersive Peclet number, Ntu and number of plates. The realistic values have been chosen for Ntu and heat capacity ratio. Since the objective of the present analysis is to bring out a new technique for transient simulation of multipass plate heat exchangers, instead of bringing out temperature responses for different type of inputs only response to step input has been calculated for different combinations of parameters. This brings out the nature of influence of these parameters on the transient response. Before going to the parametric study care has been taken so that no computational error creep into the solution during numerical Laplace inversion (because otherwise the solution is analytical and hence free from error). To ensure this the present formulation has been used to predict the gain of 1-3 pass (with seven channels) and 1-4 pass (with eight channels) heat exchangers with plug flow ($Pe \rightarrow \infty$) and absence of phase lag effect. These particular cases were presented by Masubuchi and Ito (case 7b and 8a of Ref. 13). For uniformity, here the cases are chosen in such a way that the first two passages are at counter current. The steady state gain was found in [13] to be 0.679 and 0.685, respectively. With the present model the corresponding gain values were computed to be 0.67906 and 0.68502. This shows the excellent accuracy of the computational technique used in the present simulation. Generally speaking, axial dispersion is found to deteriorate the performance of a multipass plate heat exchanger. This means a higher cold fluid and a lower hot fluid temperature at the outlet during steady state operation with the increase of dispersive Peclet number (i.e., decreasing axial dispersion). However, how this steady state is arrived at, depends critically on the pass arrangement as well as the extent of axial dispersion. It is evident from Fig. 9 that for 1–2 pass arrangement there is a cross over of response curves at different Peclet number which indicates different time rate of response i.e., the response with higher Peclet number is faster resulting in steeper response curve. On the contrary for 2–2 exchanger the transient slope does not alter much with dispersion hence the effect of dispersion is primarily the increase in initial delay period (discussed later) and effectiveness of the heat exchanger.

In all these simulations the geometrical parameters have been chosen from experience of real plate heat exchangers. For example the plate spacing has been chosen as 2% of the effective length of the plate. The entry length of the heat exchangers from the port inlet to the first plate entry has been chosen to be 10% of the effective length of the plates.

It should be mentioned here that for the present case a wide range of the values of dispersive Peclet number has been used to show its influence on the transient response. In practical applications the particular range of Peclet number can be obtained from transient experiment on a particular plate heat exchanger. For example, for a range of small welded plate heat exchangers, the Peclet number was found to be of the order of 8–10 in [18]. For another set of gasketted plate heat exchangers of larger plate aspect ratio, from both heat and mass transfer experiments the Peclet number was measured to be between 15 and 25 [19]. In some cases the Peclet number can be as low as 3.5 as found by Das et al. [16]. The dispersive Peclet number in reality depends on distribution of velocity and temperature of the fluid which in turn depends on flow parameters such as Reynolds and Prandtl number and geometrical parameters such as plate aspect ratio, corrugation geometry etc.

8.1. 1–2 pass plate heat exchangers

With the values of $R_w = 0.2$, $R_w = 1.0$, $R_{g2} = 1.0$, $R_2 = 2.0$, the overall transient response for the 1–2 pass plate heat exchangers are depicted in Figs. 7–9 depicts the responses of the heat exchangers HX1 which is first solved independent of HX2. The response shows the characteristics identical to that observed by Das et al. [16]. It can be mentioned here that the dependence of initial delay period on Peclet number is observable only in the hot fluid, because of strong phase lag effect at the entry to the hot side. A similar delay is absent in the cold side because of the fact that the cold fluid enters HX1 with zero temperature. It is also observable that the thermal performance of this heat exchanger is critically dependent on Peclet number and with lower Peclet number (i.e., higher amount of axial heat dispersion) the response of hot side becomes higher and that for the cold side lower indicating a decrease in effectiveness for the heat exchanger. The responses of the cold side depicted in this curve has been approximated by a first order system as indicated by Eq. (45) and has been used as an inlet temperature to heat exchanger HX2. The responses of which have been shown in Fig. 8. It is interesting to note here that both for cold and hot side the initial delay period of the responses of HX2 are strong functions of Peclet number which is in contrast with Fig. 7. This can be attributed to the fact that here both cold and hot side inlet of heat exchanger entering with nonzero temperatures. It is even more important to note that the slopes of the responses in the transient regime of this heat exchanger (HX2) is also critically dependent of the dispersive effect indicated by dependence of responses on the Peclet number.

The combined overall transient response of 1–2 Plate heat exchangers is shown in Fig. 9. It can be referred from Fig. 8 that the cold side outlet temperature of the whole heat exchanger is same as

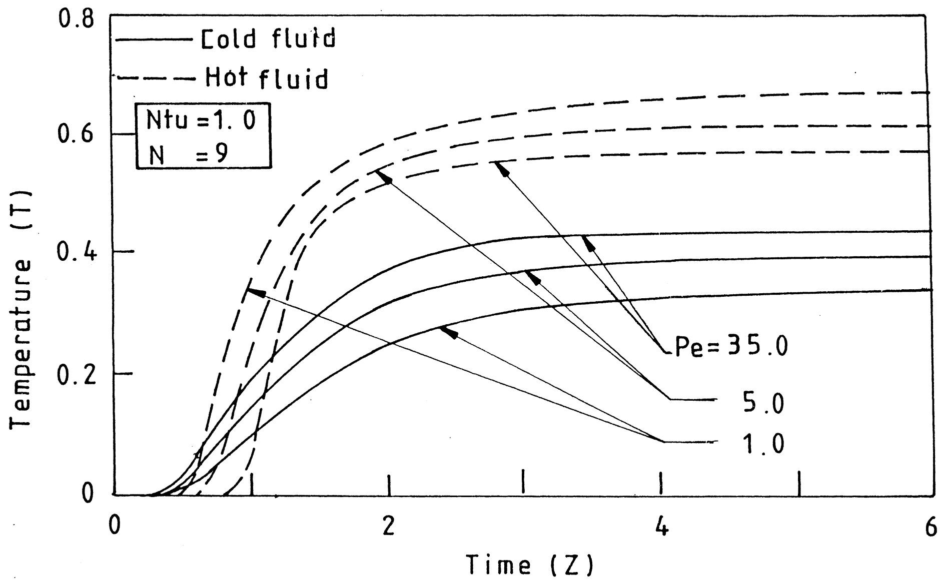


Fig. 7. Effect of Peclet number on the response of HX1 due to step change of inlet temperature in hotside fluid $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0, f_1(Z) = 0, f_2(Z) = 1.0$, Ntu $= 1.0$, $N = 9$, $l_1 = 0.10$, $l_{i+1} - l_i = 0.02$.

cold side outlet temperature of HX2 which indicates the very strong influence of Peclet number on initial delay period, time constant and gain. The gain in this case indicates effectiveness of heat exchangers in steady state which is dependent on the dispersive Peclet number. The hot side response have been calculated from the hot side responses of all channels of HX1 and HX2 by taking proper phase lag from the channel outlet. The results show very strong dependence of all the transient characteristic and steady state performance on dispersive Peclet number. The effect of Ntu on the overall performance of a 1–2 pass plate heat exchanger is shown in Fig. 10. It is observed that since the number of transfer units characterises heat transfer within channel and is not related to the phase lag, the initial delay period of response of both hot and cold side are independent of Ntu. However, the response

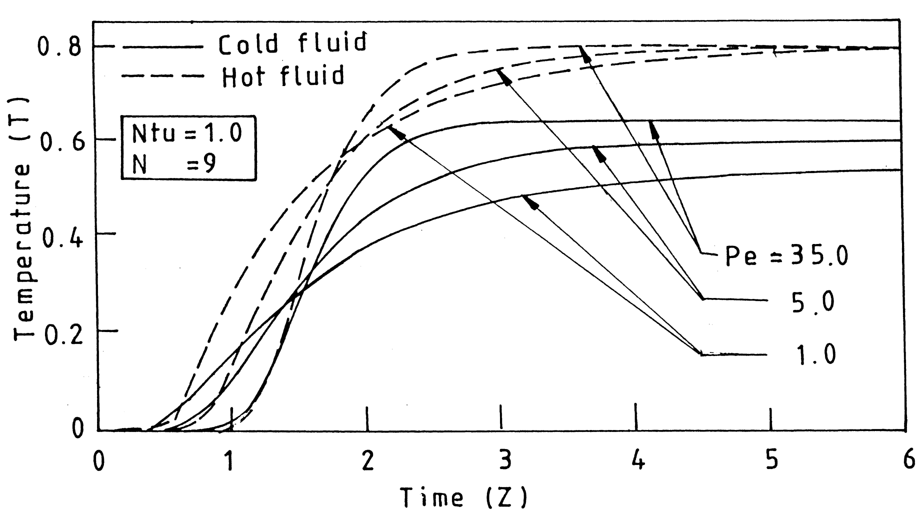


Fig. 8. Effect of Peclet number on the response of HX2 due to step change of inlet temperature in hotside fluid $R_w = 0.02$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0, f_1(Z) = 0, f_2 = 1.0$, Ntu $= 1.0$, $N = 9$, $l_1 = 0.10$, $l_{i+1} - l_i = 0.02$.

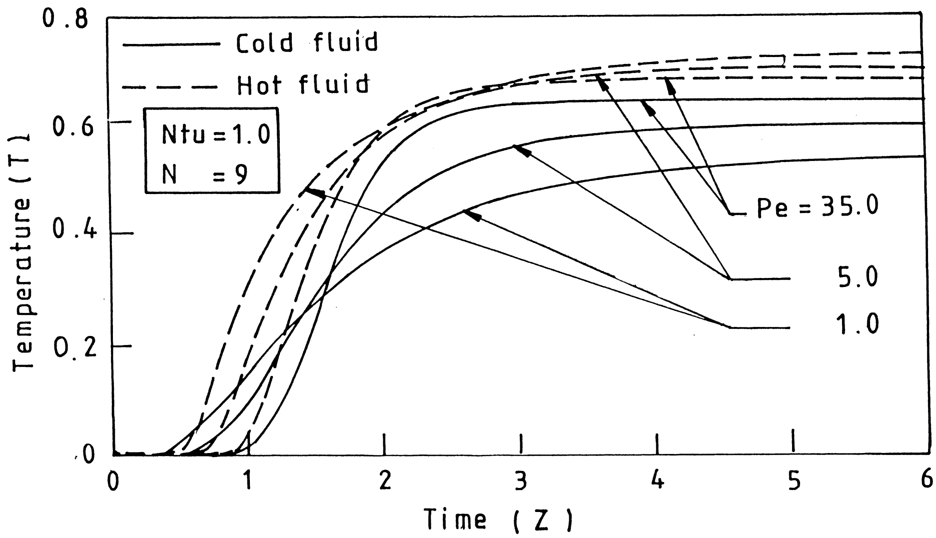


Fig. 9. Effect of Peclet number on the net response of 1–2 pass plate heat exchangers due to step change of inlet temperature in hotside fluid $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0$, $f_2(Z) = 1.0$, Ntu $= 1.0$, $N = 9$, $l_1 = 0.10$, $l_{i+1} - l_i = 0.02$.

characteristic for both transient and steady state are strongly dependent on Ntu with highest Ntu giving highest effectiveness indicated by minimum hot side and maximum coldside outlet temperature.

The effect of number of plates on the performance of this plate heat exchanger can be observed from Fig. 11. These results have been calculated with plate spacings of 5% of effective flow length of plate. It has been found that the effect of number of plates is strongly dependent on the plate spacing and with plate spacing as low as 2% of effective length the effect of number of plates are very marginal in the transient regime. With 5% of plate spacing it is observed that the transient responses are considerably delayed with increasing plate numbers. The higher number of plates provides higher efficiency. This is mainly due to the phase lag effect which increases with number of plates and the reduction of end effect with number of plates. Infact as has been identified by Khandlikar and Shah [8], the end effect ceases as number of plates approaches 40. What is important to note from the present result is, eventhough the steady state perform-

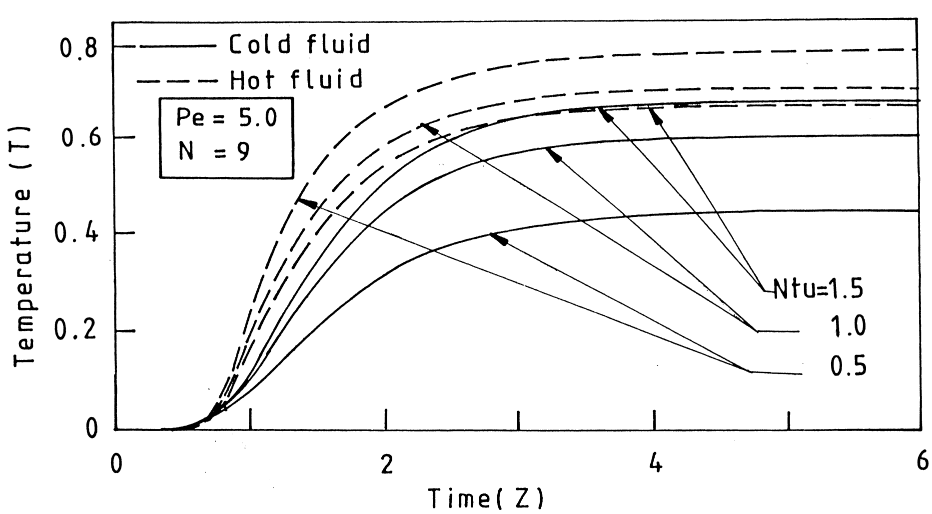


Fig. 10. Effect of Ntu on the net response of 1–2 pass plate heat exchangers $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0$, $f_2(Z) = 1.0$, $Pe = 5.0$, $N = 9$, $l_1 = 0.10$, $l_{i+1} - l_i = 0.02$.

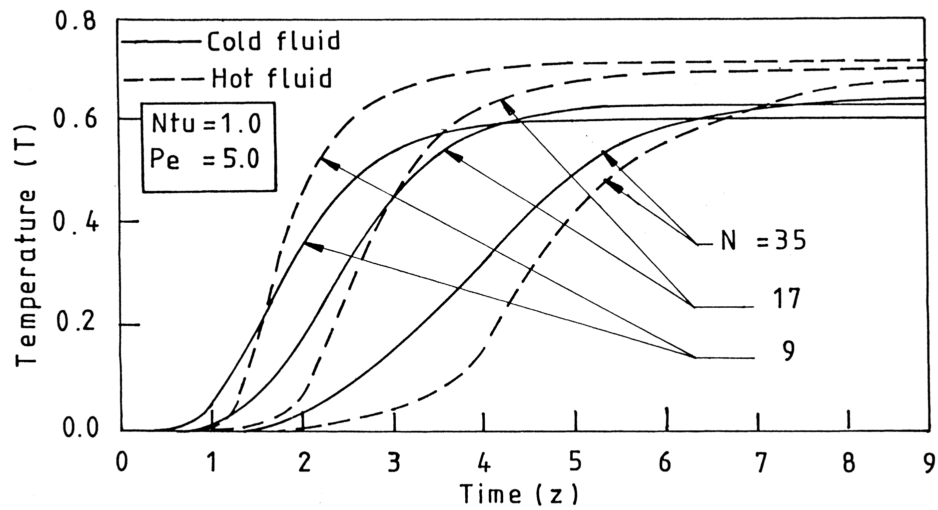


Fig. 11. Effect of number of plates on the net response of 1–2 pass plate heat exchangers $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0$, $f_2(Z) = 1.0$, Ntu $= 1.0$, $Pe = 5.0$, $l_1 = 0.10$, $l_{i+1} - l_i = 0.05$.

ances do not improve significantly from 17 to 35 plates, its transient response shows significant variation which should be taken into consideration for framing control strategies.

8.2. 2–2 pass plate heat exchangers

With the values of $R_w = 0.2$, $R_w = 1.0$, $R_g = 1.0$, $R_2 = 1.0$, the results of 2–2 pass heat exchanger are depicted in Figs. 12 and 13 which brings out the overall temperature response of 1–2 plate heat exchangers. It is interesting to observe that similar to 1–2 pass the delay time of hot fluid only is found to depend on the dispersion effect, eventhough the reason for this cannot be identical as the 1–2 pass case. Here both the hot and cold fluid temperatures of both heat exchanger HX1 and HX2 are non-zero at entry. In spite of this, during the initial temperature rise in HX1 it acts essentially similar as that of the HX1 in case of 1–2 pass type. Only after the response from HX2 is fed to HX1 its response characteristic changes. This change in characteristic of the response is depicted in Fig. 13 by the wavy nature of cold response of HX which shows definite change in response character after a certain period. However, with increasing dispersion, since the temperature profile smoothens within the fluid, the

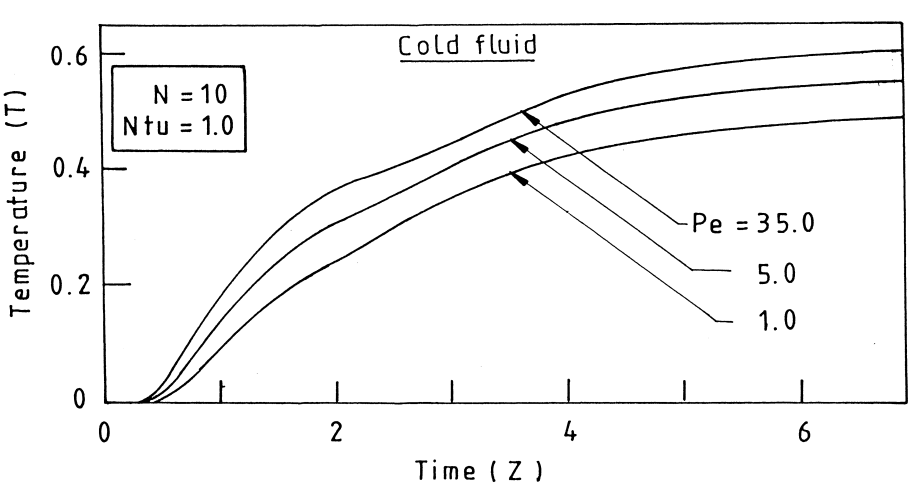


Fig. 12. Effect of Peclet number on cold fluid response of 2–2 pass plate heat exchangers $R_w = 0.02$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0$, $f_2(Z) = 1.0$, Ntu $= 1.0$, $N = 9$, $l_1 = 0.10$, $l_{i+1} - l_i = 0.02$.

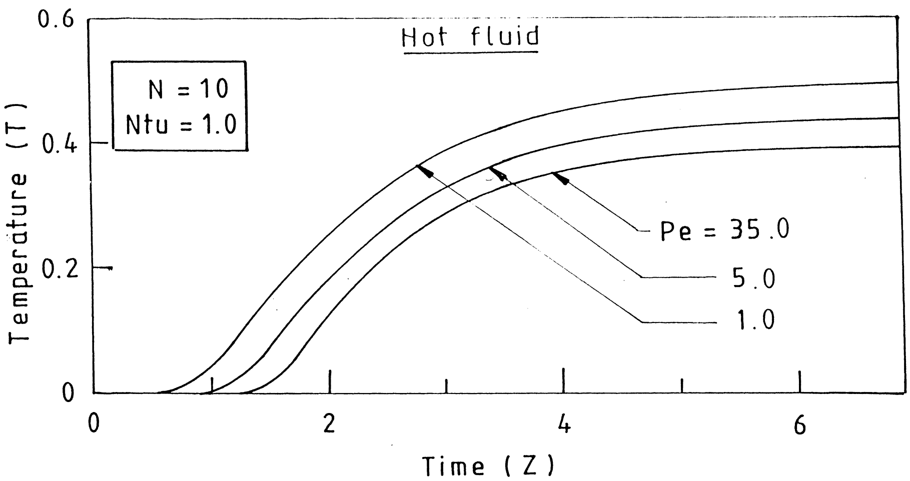


Fig. 13. Effect of Peclet number on hot fluid response of 2–2 pass plate heat exchangers $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0, f_2(Z) = 1.0$, Ntu $= 1.0$, $N = 9$, $l_i = 0.10$, $l_{i+1} - l_i = 0.02$.

sudden change in slope of response becomes less and less observable.

At this point the iterative process to be employed for solution by successive approximation of responses of HX1 and HX2 can be examined. Convergence of the successive iterations can be examined for convergence of hot response of HX1 and cold response of HX2 approximated by first order functions with delay period τ_d, time constant τ_c, and gain K. When by successive iterations these constants for both fluid converge the solution is assumed to be obtained. The typical set of such convergence data for successive iterations have been given in Table 1 which takes only few iteration for convergence.

Table 1
Convergence of response iterations (Ntu $= 1.0$, $N = 10$, $R_{g2} = 1$)

	No. of iterations	Hot outlet temperature of HX1			Cold outlet temperature of HX2		
		τ_d	τ_c	K	τ_d	τ_c	K
Pe = 1.0							
	1	0.27661	0.73690	0.66575	1.27315	1.00675	0.22050
	2	0.43089	0.93008	0.73674	1.37470	1.10020	0.24201
	3	0.43422	0.96355	0.74274	1.38273	1.11246	0.24359
	4	0.43369	0.96677	0.74312	1.38321	1.11350	0.24368
	5	0.43365	0.96698	0.74314	1.38325	1.11354	0.24368
	6	0.43365	0.96698	0.74314	1.38325	1.11354	0.24368
Pe = 5.0							
	1	0.37321	0.63476	0.61123	1.22852	0.88785	0.23707
	2	0.56782	0.86520	0.70201	1.35990	1.01932	0.27070
	3	0.58023	0.92286	0.71377	1.37647	1.04548	0.27464
	4	0.57997	0.93175	0.71501	1.37823	1.04906	0.27503
	5	0.57980	0.93283	0.71512	1.37839	1.04947	0.27506
	6	0.57980	0.93283	0.71512	1.37839	1.04947	0.27506
Pe = 35.0							
	1	0.44115	0.53739	0.56602	1.19336	0.76874	0.24557
	2	0.66812	0.79476	0.67205	1.34630	0.94156	0.29058
	3	0.68777	0.88230	0.69040	1.37199	0.98826	0.29780
	4	0.68815	0.90065	0.69309	1.37593	0.99697	0.29878
	5	0.68785	0.90377	0.69343	1.37648	0.99836	0.29890
	6	0.68785	0.90377	0.69343	1.37648	0.99836	0.29890

The effect of Ntu on the overall hot and cold pass responses of the 2–2 type plate heat exchangers have been shown in Fig. 14. It is observed that as Ntu increases the effectiveness of heat exchanger also increases and the nature of the cold side response shows some sudden change in slope due to the reasons stated above. It is to be noted here that not only the steady state effectiveness of heat exchanger depends on Ntu but its transient response also shows significant variation with it. Even though the initial delay period of the cold response is independent of Ntu, the same for the hot response appears to be Ntu dependent. This seems to be in contradiction with the expectation that the initial delay should be independent of Ntu since Ntu has got nothing to do with the delay in the port. However, on closer observation it is found that the physical appearance of the curves does not depict real picture here. Since, the curves takes care of numerical only to the second place of decimal, hence small response of the temperature at lower Ntu is difficult to recognize in the figure. The numerical values show that the temperature starts responding simultaneously i.e. independent of Ntu. Only the difference in the magnitude of temperature rise (which is dependent on Ntu) is manifested as delay. This can be confirmed from Table 2 which shows an identical delay period for all the Ntu values.

The influence of number of plates has been observed in Fig. 15. It has been found that both hot and cold responses are considerably influenced by number of plates. It should be noticed here that the hot side fluid temperature shows an increased delay with increasing number of plates which is due to the increased amount of phase lag of the hot fluid at the entry. This figure has been plotted on the basis of plate spacing of 2% of effective flow length.

9. Conclusions

An extensive analysis has been presented for the prediction of temperature transients for multipass plate heat exchangers so as to take care of fluid backmixing and flow maldistribution into consideration. The method is simple and efficient. This general method can be utilised for prediction of 1–*n* and *m*–*n* or *n*–*n* pass plate heat exchangers. The specific examples presented show a straight forward successive solution procedure for 1–*n* type plate heat exchangers where the modules of single pass (parallel and counter) plate heat exchangers to which the whole heat exchangers has been divided can be solved progressively from one end to the other. This technique, however, fails to predict solution for *m*–*n* or *n*–*n* type plate heat exchanger where the input for successive modules mutually influence the performance of the neighbouring module. As a consequence the problem reduces to be a conjugate one. This situation can be handled by iterative process. The main thrust of the present analysis lies in bringing the maldistribution effect through axial dispersion in fluid characterised by a dispersive Peclet number. The solution to the individual modules have been obtained by forming a set of partial differential equations and

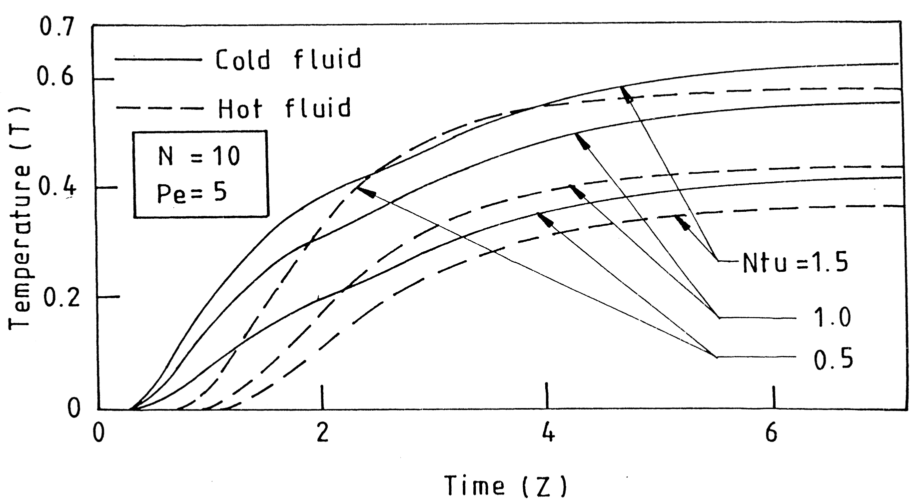


Fig. 14. Effect of Ntu on the overall response of 2–2 pass plate heat exchangers $R_w = 0.02$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0, f_2(Z) = 1.0$, $Pe = 5.0$, $N = 9$, $l_i = 0.10$, $l_{i+1} - l_i = 0.02$.

Table 2
Effect of Ntu on response delay ($R_{g2} = 1.0$, $N = 10$)

Time (Z)	Hot outlet temperature of HX1			Cold outlet temperature of HX2		
	Ntu = 0.5	Ntu = 1.0	Ntu = 1.5	Ntu = 0.5	Ntu = 1.0	Ntu = 1.5
0.0	0	0	0	0	0	0
0.1	0	0	0	0	0	0
0.2	0	0.000001	0.000002	0.000001	0	0
0.3	0.000384	0.001299	0.002506	0.000009	0.000002	0.000001
0.4	0.003012	0.008949	0.015633	0.000013	0.000009	0.000004
0.5	0.009010	0.024113	0.039182	0.000015	0.000011	0.000006
0.6	0.018505	0.045623	0.070562	0.000048	0.000055	0.000007
0.7	0.030814	0.070707	0.104749	0.001211	0.000097	0.000007
0.8	0.044746	0.096402	0.137860	0.006723	0.000155	0.000011
0.9	0.059478	0.121504	0.168867	0.019590	0.000188	0.000015
1.0	0.074485	0.145536	0.197599	0.038911	0.001984	0.000018
1.1	0.089445	0.168327	0.224121	0.064223	0.007703	0.000544
1.2	0.104136	0.189802	0.248542	0.093345	0.018329	0.003166
1.3	0.118394	0.209898	0.270958	0.124633	0.033340	0.009289
1.4	0.132078	0.228608	0.291456	0.156747	0.051622	0.018999
1.5	0.145066	0.245861	0.310108	0.188714	0.072013	0.031617

solving them after Laplace transformation. The final response in the time domain has been obtained by numerical inversion of Laplace transform using Fourier series approximation.

The plate heat exchangers are different from shell-and-tube heat exchangers with respect to dynamic response because of phase lag at the entry and the successive channels. This leads to a delay which increases even more with the increase of number of plates due to decrease of fluid velocity in the port which carries fluid to the channels. Additional phase lags are introduced between delay which increases even more with the increase of number of plates due to decrease of fluid velocity in the port which carries fluid to the channels. Additional phase lags are introduced between the passes where the fluid from the previous pass gets mixed. For the first time an exhaustive account of these phase lag including mixing of the fluid between passes has been taken into consideration in the solution algorithm. In the solution procedure, a first order approximation is made for step response which is input for successive passes. It is important to ob-

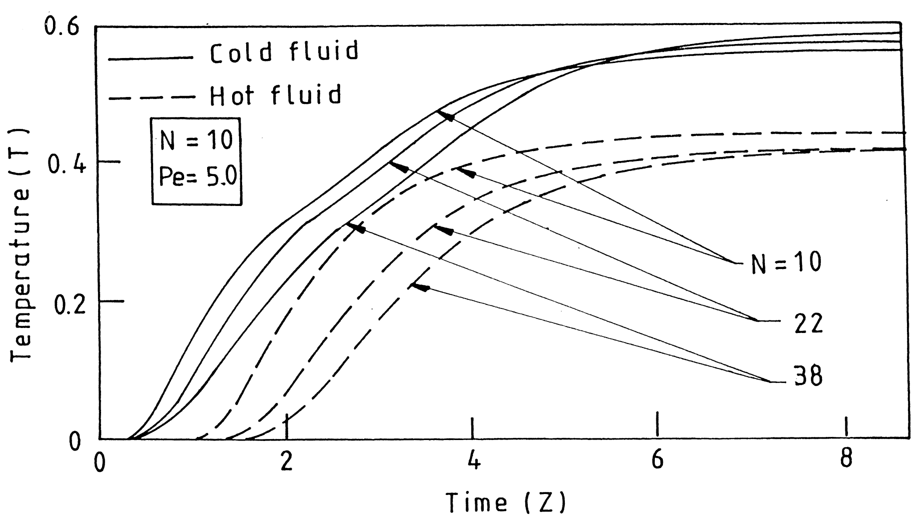


Fig. 15. Effect of number of plates on the overall response of 2–2 pass plate heat exchangers $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(Z) = 0$, $f_2(Z) = 1.0$, Ntu $= 1.0$, $Pe = 5.0$, $l_i = 0.10$, $l_{i+1} - l_i = 0.02$.

serve that the flow maldistribution results in the degradation of thermal performance of heat exchanger and parametric study with respect of number of plates and Ntu for the two types multipass arrangement have been presented. The results of the analysis clearly indicates the necessity of incorporating the dispersion and phase lag effects for the prediction of transient behaviour of multi pass plate heat exchangers.

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